

$$\textcircled{1} \quad \sum (-1)^{n+1} = 1 - 1 + 1 - 1 + \dots$$

$$1 - \sum (-1)^{n+1} = 1 - (1 - 1 + 1 - 1 + \dots)$$

$$\Rightarrow 1 - \sum (-1)^{n+1} = \sum (-1)^{n+1}$$

$$\Rightarrow 1 = 2 \sum (-1)^{n+1}$$

$$\Rightarrow \sum (-1)^{n+1} = \frac{1}{2} \quad (*)$$

$$\textcircled{2} \quad \begin{aligned} \sum (-1)^{n+1} \cdot n &= 1 - 2 + 3 - 4 + \dots \\ + \sum (-1)^{n+1} \cdot n &= 1 - 2 + 3 - 4 + \dots \end{aligned}$$

$$2 \sum (-1)^{n+1} \cdot n = 1 - 1 + 1 - 1 + \dots$$

$$\Rightarrow 2 \sum (-1)^{n+1} \cdot n = \sum (-1)^{n+1}$$

$$\Rightarrow 2 \sum (-1)^{n+1} \cdot n = \frac{1}{2}$$

$$\Rightarrow \sum (-1)^{n+1} \cdot n = \frac{1}{4} \quad (**)$$

$$\sum n = 1 + 2 + 3 + 4 + 5 + 6 + \dots$$

$$- \sum (-1)^{n+1} \cdot n = 1 - 2 + 3 - 4 + 5 - 6 + \dots$$

$$\sum n - \sum (-1)^{n+1} \cdot n = 4 + 8 + 12 + \dots = 4(1 + 2 + 3 + \dots)$$

$$\Rightarrow \sum n - \sum (-1)^{n+1} \cdot n = 4 \cdot \sum n$$

$$\Rightarrow - \sum (-1)^{n+1} \cdot n = 3 \sum n$$

De (**):

$$- \frac{1}{4} = 3 \sum n$$

$$\Rightarrow \boxed{\sum n = -\frac{1}{12}}$$